# Numerical Study on Laminar Flow over Three Side-by-Side Cylinders

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The present study has numerically investigated two-dimensional flow over three circular cylinders in an equidistant side-by-side arrangement at a low Reynolds number. For the study, numerical simulations are performed, using the immersed boundary method, in the range of $g^* < 5$ at $Re = 100$, where $g^*$ is the spacing between two adjacent cylinder surfaces divided by the cylinder diameter. Results show that the flow characteristics significantly depend on the gap spacing and a total of five kinds of wake patterns are observed over the range: modulation-synchronized ($g^* \gtrsim 2$), inphase-synchronized ($g^* \approx 1.5$), flip-flopping ($0.3 < g^* \lesssim 1.2$), deflected ($g^* \approx 0.3$), and single bluff-body patterns ($g^* < 0.3$). Moreover, the parallel and symmetric modes are also observed depending on $g^*$ in the regime of the flip-flopping pattern. The corresponding flow fields and statistics are presented to verify the observations.

**Key Words:** Side-by-Side Arrangement, Modulation-Synchronized, Inphase-Synchronized

## Nomenclature

$C_D$, $C_L$ : drag and lift coefficients
$d$ : cylinder diameter
$g$, $g^* (= g/d)$ : gap spacing
$M$, $N$ : spatial resolution
$p$ : pressure
$Re$ : Reynolds number
$St$ : Strouhal number
$t$ : time
$u_i$, $(u, v)$ : velocity components
$u_\infty$ : free-stream vlocity
$x_i$, $(x, y)$ : Cartesian coordinates
$\Delta$ : increment

### Superscripts

$\overline{\phantom{x}}$ : time-average
$'$ : fluctuation

### Subscripts

$rms$ : root mean square
$i$ : indices (1, 2, 3)

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## 1. Introduction

Vortex shedding occurs in the near wake behind a single bluff body or multiple ones due to the flow instability, resulting in fluctuating flow fields and thus drag and lift forces. Such fluctuating forces may cause structural vibrations, acoustic noise or resonance, which in some cases can trigger structure failure or enhance mixing in the wake. Accordingly, bluff-body flows have long drawn a great many concerns in the field of fluids engineering, in view of their academic and engineering importance. Most flows of engineering interests involve a group of multiple bluff bodies, rather than a single bluff body, and, depending on the number and arrangement of the constituent bodies, mutual interactions among their wakes may be very crucial. Nevertheless, only a few studies have been so far performed on flow over multiple bluff bodies, which motivates the present study.

The present study is concerned with flow over multiple, especially three, circular cylinders of equal diameter and gap spacing in a side-by-side arrangement. Despite its relatively simple configuration, the flow can be found in many engineering applications, for example tube bundles in heat exchangers, fuel and control rods in nuclear reactors, piers and bridge pilings, oil and gas pipelines, cooling-tower arrays, suspension bridges and high-rise buildings: refer to Zhou et al. (2000) for more details. The flow, characterized by mutual interactions among multiple basic single-cylinder wakes, is governed by two nondimensional flow parameters; one is the Reynolds number $Re=u_\infty d/\nu$ and the other is the nondimensional gap spacing $g^*=g/d$, where $u_\infty$ is the free-stream velocity, $d$ the cylinder diameter, $g$ the gap spacing between two adjacent cylinder surfaces, and $\nu$ the kinematic viscosity. It is well known that for $g^*\gtrsim 5$ the mutual interference disappears, leading to separate single-cylinder wakes.

Since it is the most fundamental, investigations on flow over multiple side-by-side circular cylinders have been mainly focused on the two-cylinder configuration, for example experimentally by Williamson (1985), Kim and Durbin (1988), Sumner et al. (1999), and Zhou et al. (2002), and numerically by Kang (2003). Such investigations have contributed to much improved understanding of flow over multiple bluff bodies.

In the meantime, the two-cylinder flow bears similarity to flow over more-than-two side-by-side circular cylinders, in many aspects such as synchronization and merging of wakes, deflection and flip-flopping of gap flows, and narrow-and-wide structures. However, the two-cylinder flow may not be representative of flows over multiple bluff bodies because both the flows exhibit *quite disparate* behaviors: refer to Sumner et al. (1999) and Zhang and Zhou (2001) for more details. The relevant studies on flow over more-than-two cylinders are summarized in Table 1, implying that the flow has received even less attention. Evidently, such previous studies can be largely divided into two branches, based on the number of the constituent cylinders $n$: small group of cylinders ($n=3\sim5$) and large group ($n=7\sim\infty$).

The present study pays attention to flow over three circular cylinders in a side-by-side arrangement. Apparently, this flow is more complicated than the two-cylinder flow: for example, there are two gap flows that may deflect in various ways, consequently involving more types of wake patterns (Zhang and Zhou, 2001). In addition, the flow is more representative of flows over multiple bluff bodies of engineering interests. Following the early study of Eastop and Turner (1982), only a few experimental researches have been performed on flow over three side-by-side circular cylinders. Kumada et al. (1984) experimentally investigated the flow at $Re=(1.0-3.2)\times10^4$ by measuring the pressure and velocity distributions and vortex-shedding frequencies and then identified four quasi-stable wake-pattern regimes, mainly depending on the gap spacing. It

**Table 1** Previous investigations of flows over more-than-two side-by-side circular cylinders of equal diameter and gap spacing. E, experimental; N, numerical; n, number of cylinders; ∞, unspecified

| Researchers | E/N | $n$ | $Re(\times10^3)$ | $g^*$ |
|---|---|---|---|---|
| Ishigai & Nishikawa (1975) | E | 5 | 33-40 | 0.2-1.5 |
| Eastop & Turner (1982) | E | 3 | 45-111 | 1.2-2.6 |
| Kumada et al. (1984) | E | 3 | 10-32 | 0-2.75 |
| Moretti & Cheng (1987) | E | ∞ | 2.5 | 1.3 |
| Zdravkovich & Stonebanks (1990) | E | 7, 11 | 37-74 | 1.1-1.75 |
| Le Gal et al. (1996) | E | 11, 21 | 0.1 | 0.5, 2 |
| Guillaume & LaRue (1999) | E | 3 | 2.5, 4.4 | 0.338-1.202 |
| | | 4 | 2.5 | 0.338-1.202 |
| Sumner et al. (1999) | E | 3 | 0.5-3 | 0-2 |
| Zhou et al. (2000) | E | 3 | 1.8 | 0.5, 2 |

was observed that, in the gap-spacing range of $g^*<0.125$, the three cylinders behaved like a single bluff body and one vortex street occurred (single bluff-body flow). For $0.125<g^*<0.35$, both the gap flows were deflected towards one side, thus forming narrow and wide wakes (asymmetric-biased flow). For $0.35<g^*<1.225$, the two gap flows were deflected outwards or towards each free stream, causing two narrow wakes behind the two outer cylinders and one wide wake behind the central one (symmetric-biased flow). For $1.225<g^*<2.75$, three separate vortex streets were seen (synchronized-unbiased flow).

Subsequently, Guillaume and LaRue (1999) measured the variation of the base pressure coefficient and the characteristics of the velocity power spectra for the flow at $Re=2.5\times10^3$. Then, they observed three quasi-stable wake patterns for $0.338\leq g^*\leq0.730$, one quasi-stable pattern with forced flopping for $0.730\leq g^*\leq0.850$, and only one quasi-stable pattern for $0.850\leq g^*\leq1.202$. Recently, Sumner et al. (1999) investigated the flow in the ranges of $Re=500-3000$ and $g^*=0-2$ using flow visualization and particle image velocimetry (PIV). In the study, either single bluff-body behaviour or an asymmetric-biased flow pattern could be observed at small gap spacings $(0\leq g^*<0.2)$, whereas a symmetric-biased wake pattern was found at intermediate spacings $(0.2\lesssim g^*\lesssim1.2)$. As reviewed so far, previous studies have shown that the wake pattern of flow over three side-by-side circular cylinders depends very strongly on the gap spacing, but relatively weakly on the Reynolds number.

To our best knowledge, however, no numerical study has been so far performed on the flow (see Table 1). Since it may furnish plentiful information easily inaccessible to the experimental study, the numerical study is required for further improved understanding of the flow. Therefore, the present study aims to numerically investigate the characteristics of flow over three circular cylinders in a side-by-side arrangement, and then to further understand the corresponding underlying mechanism. For the study, we concentrate on scrutinizing the effect of the gap spacing on the flow, particularly for the purpose of identifying various kinds of wake patterns that may occur due to the mutual interactions among three basic single-cylinder wakes or between two gap flows. Numerical simulations are performed, using the immersed-boundary method developed by Kim et al. (2001) for simulating flows over or inside complex geometries. In the immersed-boundary method, both momentum forcing and mass source/sink are applied on the body surface or inside the body to satisfy the no-slip condition and continuity on or around the immersed boundary, leading to significant memory and CPU savings and easy grid generation compared to the unstructured grid method. In the present study, we deal with the flow in the ranges of $g^*<5$ at $Re=100$ that is assumed to be two-dimensional and laminar.

## 2. Numerical Method

Numerical simulations of two-dimensional unsteady incompressible flow have been conducted using the same immersed-boundary method adopted for the two-cylinder flow (Kang, 2003). The relevant governing equations are integrated in time using a second-order semi-implicit fractional-step method: a third-order Runge-Kutta method (RK3) for the convection terms and a second-order Crank-Nicolson method for the diffusion terms. In space, on the other hand, the governing equations are resolved with a finite-volume approach on a staggered mesh. Here, the Cartesian $(x, y)$ coordinate system is adopted as a basis for the application of the immersed-boundary method. More details associated with the immersed-boundary method are described in Kim et al. (2001).

The present computational domain is schematically diagramed in Fig. 1. The computational domain extends to $-50<x<30$ and $-50<y<50$ and the three circular cylinders are located with the centers at $(0, y_c)$ where $y_c=0$ and $\pm(g^*+1)$. A uniform distribution of $32\times32$ grid points is used within the cylinder diameters, whereas the tangential-hyperbolic grid distribution is in the outer regions. In the gaps, a uniform or nonuniform grid distribution is flexibly applied depen-

**Table 2** Validation of the numerical method : comparison study for flow over a single circular cylinder at $Re$=40 and 100 [$M \times N$= $389 \times 177$, $\Delta x_c (\Delta y_c)$ =0.03125 and $\Delta t$= 0.02]. Here, $\Delta x_c$ and $\Delta y_c$ denote the magnitudes of grid spacings inside and around the three cylinders

| | $Re$ | $\bar{C}_D$ | $C'_L$ | $St$ |
|---|---|---|---|---|
| Present | 40 | 1.51 | | |
| | 100 | 1.33 | 0.32 | 0.165 |
| Park et al. (1998) | 40 | 1.51 | | |
| | 100 | 1.33 | 0.33 | 0.165 |
| Williamson (1989) | 100 | | | 0.164 |

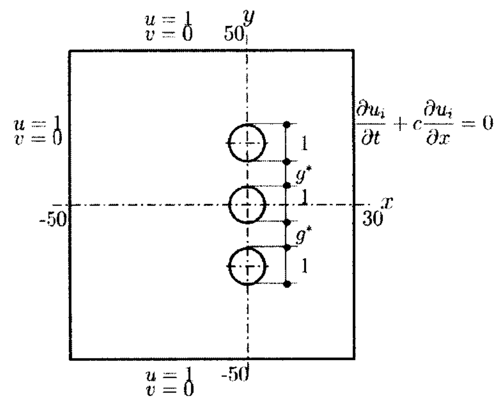


**Fig. 1** Schematic diagram of the present computational domain

ding on the gap spacing. As the gap spacing varies, the number of total grid points in the y direction is properly adjusted such that the resolution close to the cylinders is preserved. A computational time step of $\Delta t$=0.02 is used for time advancement in all the computations performed in the present study.

To confirm the accuracy, numerical simulations have been performed on flow over a single circular cylinder at $Re$=40 and 100, and their typical results are compared with other existing values in Table 2. Here, $\bar{C}_D$ and $C'_L$ are, respectively, the time-averaged drag coefficient and the maximum amplitude of the lift-coefficient fluctuations, and $St$ denotes the nondimensional vortex-shedding frequency (Strouhal number). Comparison in the table shows that the present results are in excellent agreement with those of Park et al. (1998) and Williamson (1989), certainly validating the present immersed-boundary method.

## 3. Results

After verifying the numerical method, we have conducted numerical simulations by varying the gap spacing in the range of $g^* < 5$ at a fixed Reynolds number of $Re$=100. All the flows considered in the present study are assumed to be two-dimensional and laminar even if they are not physically so for small gap spacings (about $g^* < 0.4$) at this Reynolds number.

### 3.1 Wake Patterns

Numerical simulations indicate that flow over three side-by-side circular cylinders at a low Reynolds number significantly depends on the gap spacing. Figures 2 through 7 (except 5) show the time evolutions of the lift and drag coefficients, and the corresponding instantaneous vorticity contours and streamlines in the fully-developed state at different gap spacings. Since the fully-developed flow is independent of the initial condition, all the simulations may be started with arbitrary initial conditions only if the fully-developed flow fields are to be analyzed. The figures present five disparate wake patterns observed at different gap spacings even for the same Reynolds number of $Re$=100 : in sequence, modulation-synchronized ($g^*$=3), inphase-synchronized (1.5), flip-flopping (0.8), deflected (0.3), and single bluff-body wake patterns (0.2). Such strong dependency of the wake pattern on the gap spacing agrees well to previous experimental studies performed at much higher Reynolds numbers (Eastop and Turner, 1982 ; Kumada et al., 1984 ; Guillaume and LaRue, 1999 ; Sumner et al., 1999 ; Zhou et al., 2000).

#### 3.1.1 Wake patterns at large gap spacings ($g^*$>1.2)

Figure 2 shows that, in case of $g^*$=3, the three single-cylinder wakes become synchronized, entailing fully-periodic flow. It is also shown that modulation occurs and the observed modulation period is very large as compared to the vortex-

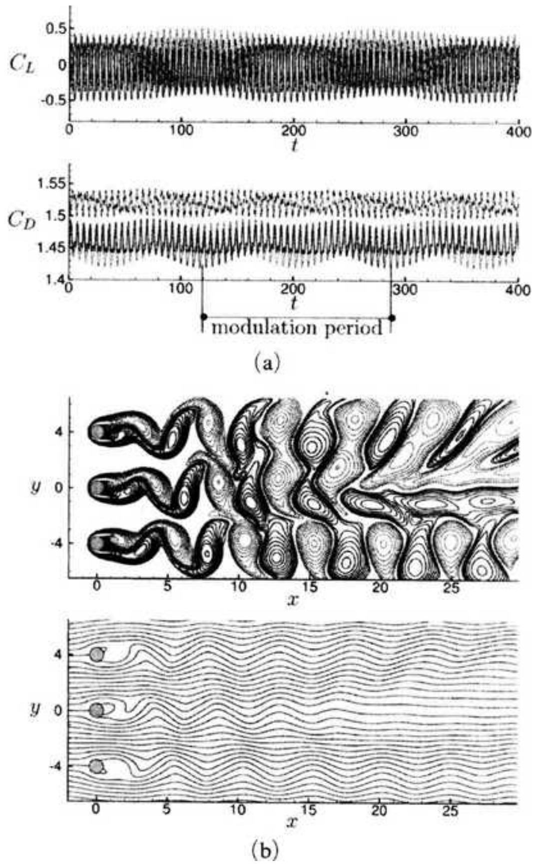


**Fig. 2** Modulation-synchronized wake pattern behind three side-by-side cylinders: (a) time evolutions of the lift and drag coefficients, and (b) instantaneous vorticity contours and streamlines at $g^*=3$ and $Re=100$. In the time evolutions, ———, lower cylinder; —··—, central; ·····, upper (also applicable to Figs. 3 through 8)

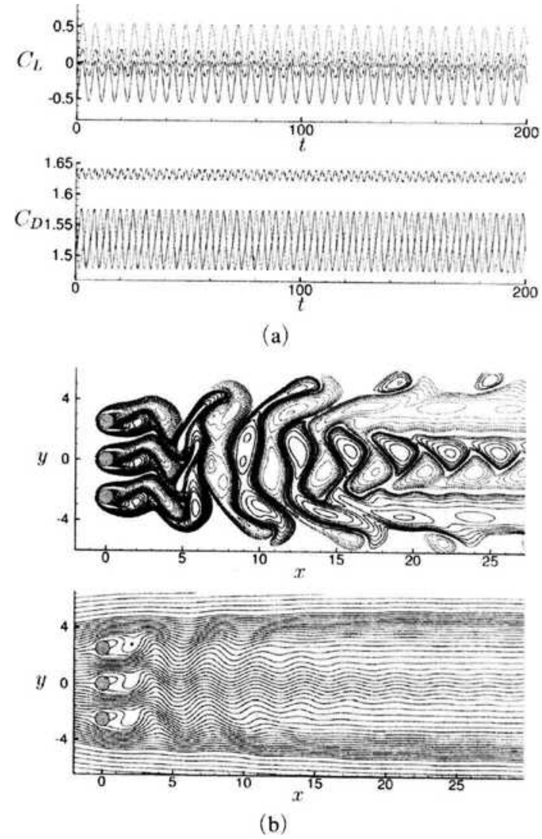


**Fig. 3** Inphase-synchronized wake pattern behind three side-by-side cylinders: (a) time evolutions of the lift and drag coefficients, and (b) instantaneous vorticity contours and streamlines at $g^*=1.5$ and $Re=100$

shedding period. According to the frequency (FFT) analyses, the three cylinders shed their vortices with frequencies close to the single-cylinder case ($St_n=0.165$), but the (same) shedding frequency for the two outer cylinders is slightly smaller than that for the central one ($St_o=0.17667$ versus $St_c=0.18250$). This minute difference between the two vortex-shedding frequencies obviously causes the modulation phenomenon with a very low frequency of $St_{mod}=St_c-St_o=0.00583=1/171.5$. A similar phenomenon is also observed in case of $g^*=4$: $St_{mod}=0.00417=1/239.8$, $St_c=0.17750$ and $St_o=0.17333$. When modulation appears, in general, the following relation can be derived:

$$St_{mod} \approx n \times St_c + m \times St_o, \qquad (1)$$

where $n$ and $m$ are integers. That is, the present modulation phenomenon corresponds to $n=1$ and $m=-1$. Thus, due to the modulation characteristics, the flow structure is called the *modulation-synchronized* wake pattern. Such wake pattern is observed for $g^* \gtrsim 2$ at the Reynolds number ($Re=100$) considered in the present study. It has been reported, on the other hand, that *synchronized-unbiased* vortex shedding occurred at the similar gap-spacing range ($g^* \gtrsim 1.5$) in the turbulent regime (Kumada et al., 1984;

Sumner et al., 1999). Thus, it is implied that the modulation phenomenon may be characteristic of the flow at low Reynolds numbers. Note that, in the configuration of two side-by-side cylinders, the bifurcation phenomenon of the antiphase- and inphase-synchronized wake patterns occurred instead of the modulation phenomenon (Kang, 2003).

In case of $g^*=1.5$, as shown in Fig. 3, the three wakes also become synchronized but in a different way. That is, the lift coefficients for the three cylinders are all in phase (of the same phase), whereas the drag coefficients for the two outer cylinders are out of phase and their (same) frequency is two times that for the central one. Due to the characteristics in the lift coefficients, the flow structure is called the *inphase-synchronized* wake pattern. Moreover, vortex shedding at the three cylinders occurs in nearly the same phase, pairs of like-signed vortices merge and then some distance downstream the wake structure becomes nearly symmetric with respect to the centerline. Such inphase-synchronized wake pattern can also be observed in the two-cylinder flow (Kang, 2003). However, the possibility of its existence in the turbulent regime has not been reported in literature.

### 3.1.2 Wake patterns at intermediate gap spacings

With further reducing the gap spacing at the same Reynolds number, for example at $g^*=0.8$, the flow is no longer periodic but becomes drastically unsteady, as evidently observed in Fig. 4. Thus, the flow structure is called the *flip-flopping* wake pattern. Such wake pattern is observed in the range of $0.3<g^*\lesssim1.2$ at $Re=100$ considered in the present study. Moreover, two types of flip-flopping wake patterns are observed in the gap-spacing range: one is that the two gap flows run downstream (nearly) parallel with each other and the other is that the two gap flows run towards each free stream, leading to narrow wakes behind the two outer cylinders and a wide wake behind the central one. Obviously, the two flip-flopping patterns are illustrated respectively in Figs. 4(b) and (c). In the former pattern (parallel mode),

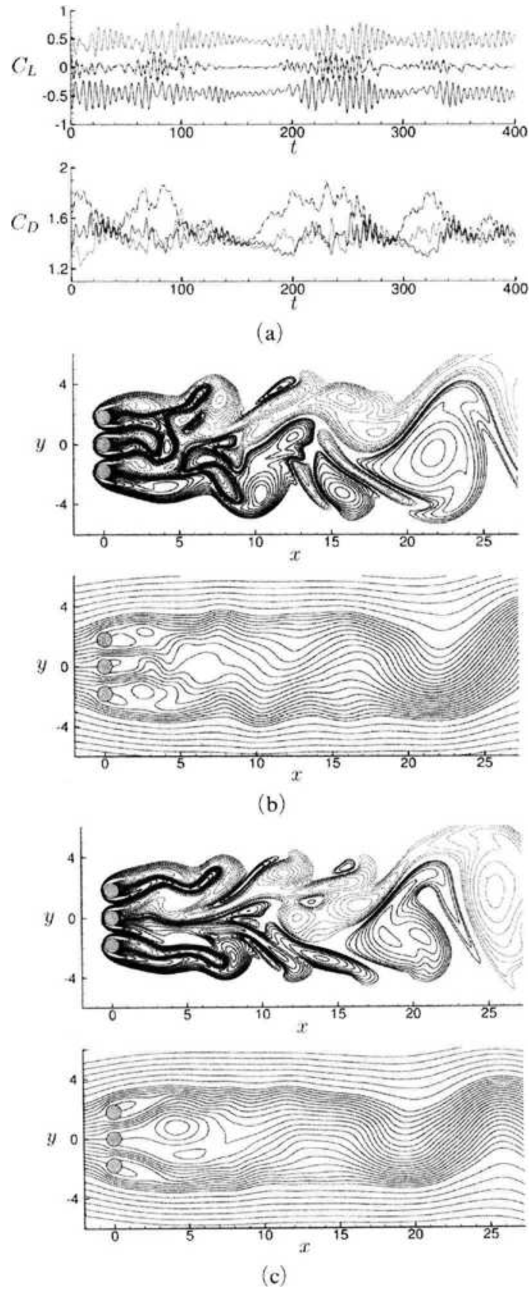


**Fig. 4** Flip-flopping wake pattern behind three side-by-side cylinders: (a) time evolutions of the lift and drag coefficients, and (b) and (c) instantaneous vorticity contours and streamlines (respectively $t=200$ and 400) at $g^*=0.8$ and $Re=100$

the drag coefficient for the central cylinder is much higher than those for the two outer cy-

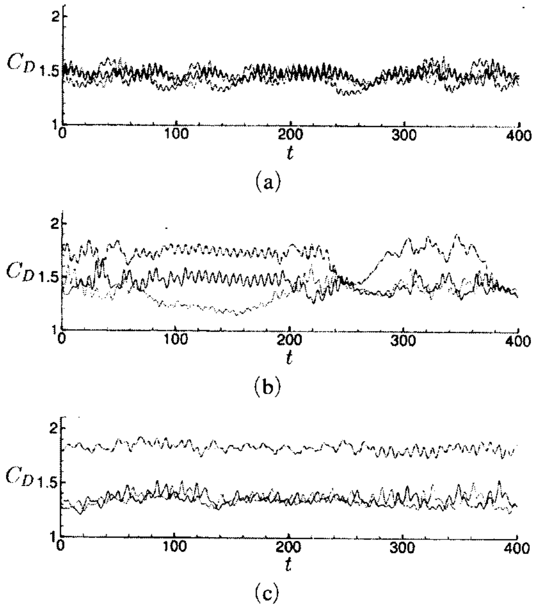


**Fig. 5** Time evolutions of the drag coefficient at $g^*=$(a) 1, (b) 0.7 and (c) 0.5 in the regime of flip-flopping wake pattern behind three side-by-side cylinders for $Re=100$

linders. In the latter pattern (symmetric mode), on the other hand, the drag coefficient for the central cylinder drops comparable to those for the two others. It is implied from Fig. 4(a) that the flow at $g^*=0.8$ and $Re=100$ intermittently flip-flops between both the parallel and symmetric modes. According to the time traces, there is no biased direction between the two outer cylinders in the sense of time averaging.

The time traces of the drag coefficient shown in Fig. 5 indicate how the flow in the flip-flopping wake pattern modifies with change of the gap spacing. In case of $g^*=1$, the three drag coefficients are comparable, indicating that only the symmetric mode occurs in the flow. In case of $g^*=$ 0.5, the drag coefficient for the central cylinder is constantly higher than the other values, indicating that only the parallel mode occurs. The two flows ($g^*=1$ and 0.5) can be considered quasi-stable in that their states persist for quite a long time. In case of $g^*=0.7$, on the other hand, both the modes occur alternately but randomly, like at $g^*=0.8$ shown in Fig. 4. Such similar phenomena are also observed in the previous experimental studies: see Sec. 1 for the literature survey. Kumada et al. (1984), Guillaume and LaRue (1999) and Sumner et al. (1999) reported that the *asymmetric-* and *symmetric-biased* flow modes occurred in the range of the flip-flopping pattern at much higher Reynolds numbers and, with increasing gap spacing, the flow changed from the former mode to the latter. Note that the two modes correspond respectively to the parallel and symmetric modes in the present study. However, there is a difference between the parallel and asymmetric-biased modes in that the gap flows for the former mode are much less deflected than those for the latter. Kumada et al. (1984) and Guillaume and LaRue (1999) claimed that the drag coefficient for the central cylinder dropped below the values for the two outer cylinders when the symmetric-biased mode occurred in the turbulent regime. However, the three drag coefficients are comparable at such low Reynolds number ($Re=100$) considered in the present study.

In case of $g^*=0.3$, as shown in Fig. 6, the flow becomes again periodic. Simultaneously, both the gap flows are deflected towards one of the two free streams and the deflection way does not change any longer. Out of the two outer cylinders, the cylinder at the side towards which the gap flows are deflected invariably experiences the higher drag coefficient, although their values are much lower than that for the central one. One vortex [of negative sign in Fig. 6(b)] at the gap side of the higher-drag outer cylinder, as well as two vortices shed from the central cylinder, are wholly suppressed by those [of positive sign in Fig. 6(b)] shed from the two outer cylinders. Downstream only one street composed of large-scale vortices of alternate positive and negative signs is formed. As well, the deflection of the gap flows results in the structure of one narrow and one wide wakes behind the three cylinders, which can be similarly observed in the configuration of two side-by-side cylinders (Kang, 2003). Due to such invariable deflection way, the flow structure is called the *deflected* wake pattern. However, the wake pattern has not been observed in the experimental studies performed at much higher Reynolds numbers.

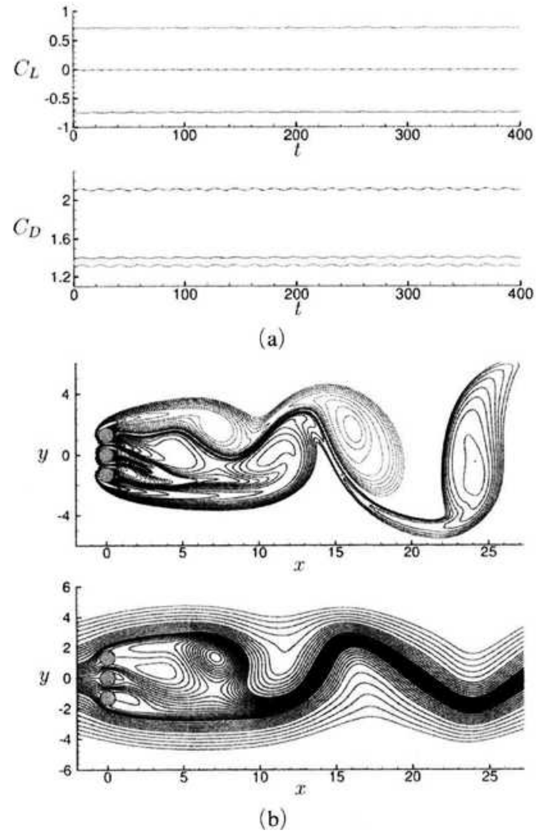


**Fig. 6** Deflected wake pattern behind three side-by-side cylinders : (a) time evolutions of the lift and drag coefficients, and (b) instantaneous vorticity contours and streamlines at $g^*=0.3$ and $Re=100$

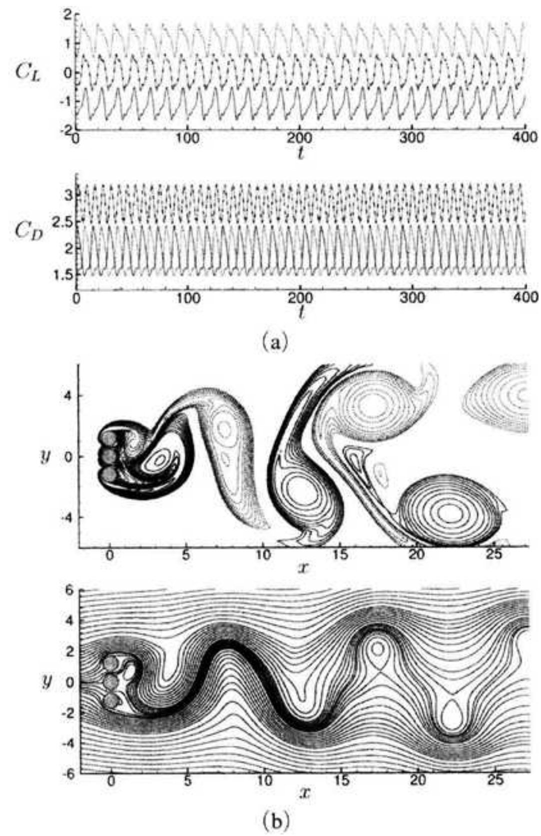


**Fig. 7** Single bluff-body wake pattern behind three side-by-side cylinders : (a) time evolutions of the lift and drag coefficients, and (b) instantaneous vorticity contours and streamlines at $g^*=0.2$ and $Re=100$

#### 3.1.3 Wake patterns at small gap spacings ($g^*<0.3$)

Finally, with three cylinders in very close proximity, the flow behaves as if there exists just one single bluff body but with an augmented characteristic length. As shown in Fig. 7 for $g^*=0.2$, the two gap flows are too weak to affect the wake pattern, leading to the complete suppression of all the vortices shed at the gap sides of the three cylinders (inner vortices). Thus, the flow structure is called the *single bluff-body* wake pattern. One street of vortices shed at the free-stream sides (outer vortices), which is formed in the near-cylinder region, becomes changed to two vortex streets of like signs further downstream. Such wake pattern can also be observed in the experimental studies performed at much higher Reynolds numbers (Kumada et al., 1984 ; Sumner et al., 1999).

### 3.2 Flow statistics

#### 3.2.1 Strouhal number

Table 3 shows the variation of the vortex-shedding frequencies (Strouhal numbers) with the cylinder gap-spacing for $Re=100$. As shown in Figs. 2 through 7, a total of five kinds of wake patterns occur depending on the gap spacing at the Reynolds number : single bluff-body ($g^*=0.2$), deflected (0.3), flip-flopping (0.5, 0.8 and 1), inphase-synchronized (1.5) and modulation-synchronized wake patterns (3 and 4). For all the

**Table 3** Variation of the Strouhal number with respect to $g^*$ at $Re=100$

| | | $g^*$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 0.2 | 0.3 | 0.5 | 0.8 | 1 | 1.5 | 3 | 4 |
| St | upper | 0.069 | 0.058 | (0.115) | - | (0.190) | 0.173 | 0.177 | 0.173 |
| | central | 0.069 | 0.058 | (0.115) | - | - | 0.173 | 0.183 | 0.177 |
| | lower | 0.069 | 0.058 | (0.114) | - | (0.190) | 0.173 | 0.177 | 0.173 |

wake patterns except the flip-flopping pattern, the fully-developed flows are periodic and thus only the fundamental frequencies are listed in the table.

For the flip-flopping wake pattern, on the other hand, the fully-developed flows are no longer periodic. Consequently, the Fourier analyses on the lift coefficients have to be conducted. For the analyses, the lift coefficients are collected for 1200 in nondimensional time after the fully-developed state. Results show that distinctive dominant-frequencies are irregularly scattered over a certain broad-banded frequency range, implying that multiple frequencies are intricately involved. In case that the peak value of dominant frequencies markedly exists (see $g^*=0.5$ and 1), the corresponding frequency is parenthesized in Table 3. Otherwise (see $g^*=0.8$), no frequency is written.

As shown in Table 3, the vortex-shedding frequency strongly depends on the gap spacing, especially at $g^*<1.5$. In addition, the three cylinders involve the same dominant vortex-shedding frequency, except in the range of the modulation-synchronized wake pattern (at $g^*\gtrsim 2$) where the shedding frequency for the central cylinder is slightly larger than those for the two outer cylinders. For the synchronized wake patterns, the shedding frequencies are nearly constant over the gap spacing, very close to that ($St_n=0.165$) in the single-cylinder case. As well, the shedding frequencies in the inphase case ($g^*=1.5$) are larger than those in the modulation case ($g^*\gtrsim 2$). Next, the (fundamental) frequencies drastically decrease with decreasing gap spacing for the flip-flopping pattern, reach a minimum value for the deflected pattern, and then slightly increase for the single bluff-body pattern.

### 3.2.2 Drag and lift coefficients

Figure 8 shows the variations of the time-averaged drag and lift coefficients ($\overline{C}_D$ and $\overline{C}_L$) and the rms value of the lift-coefficient fluctuations ($C'_{Lrms}$) with the gap spacing at $Re=100$. It is obviously shown that the flow quantities depend strongly on the gap spacing. The quantity values for the two outer cylinders are nearly the same except in the range of the deflected wake pattern ($g^*=0.3$), implying that the flow is symmetric with respect to the centerline in the sense of time averaging.

The drag coefficients for the three cylinders stay nearly constant, with all the values slightly larger than the single-cylinder case ($\overline{C}_{Dn}=1.33$), when the gap spacing is large. However, the mean drag for the central cylinder steeply increases with decreasing $g^*$ in the range of $g^*<1$ (the flip-flopping and single bluff-body wake patterns), whereas those for the two outer cylinders do so just in the range of $g^*<0.3$ (the single bluff-body wake pattern). Consequently, the difference in the mean drag between the central and outer cylinders rapidly increases with decreasing $g^*$ in the range of $g^*<1$. Likewise, the mean lift coefficients for the outer cylinders also depend strongly on the gap spacing while that for the central one is null.

The lower and upper cylinders have the positive and negative mean values, respectively, implying that there exist mean repulsive lift forces exerted between two adjacent cylinders. That is, the upper cylinder exerts the downward lift force on the central one, and the lower cylinder exerts the upward force with nearly the same strength. The mean lift coefficients for the two outer cylinders steadily increase in their magnitudes with decreasing $g^*$ all over the gap spacing. Note that the three-dimensional effect, which may be important at very small gap spacings, is not considered in the present study.

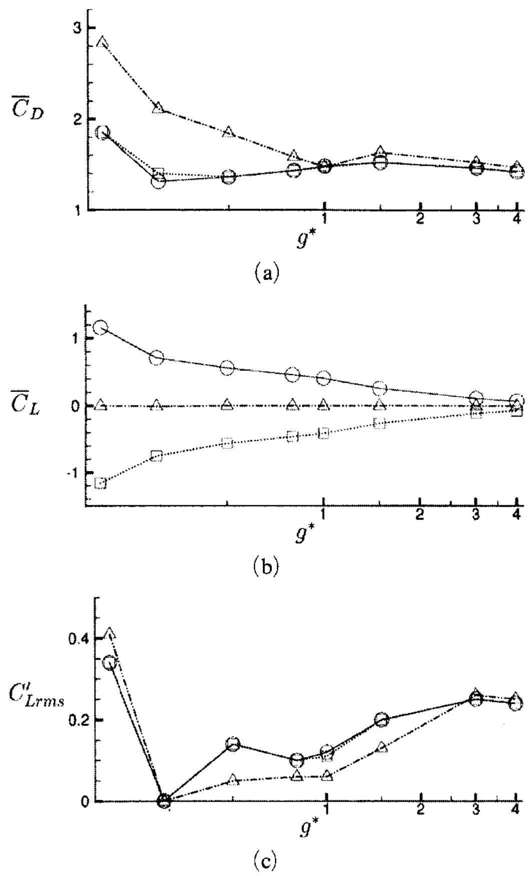


**Fig. 8** Variations of the drag and lift coefficients with respect to $g^*$ at $Re=100$: (a) time-averaged drag coefficient, (b) time-averaged lift coefficient and (c) rms value of the lift-coefficient fluctuations

On the contrary, the lift fluctuations for the three cylinders stay nearly constant, with all the values relatively smaller than the single-cylinder case ($C'_{Lrms}=0.32$), for $g^*\gtrsim 2$ in the modulation-synchronized wake pattern. Then, they all decrease with decreasing $g^*$ in the inphase-synchronized wake pattern at $g^*\approx 1.5$, with the value for the central cylinder decreasing more steeply than those for the outer ones. With further decreasing $g^*$ in the range of the flip-flopping wake pattern, subsequently, the lift fluctuation for the central cylinder remains nearly constant and those for the two outer cylinders slightly increase and then decrease. Consequently, all the lift fluctuations reach a local minimum at $g^*\approx 0.3$ in the range of the deflected wake pattern. Finally, the lift fluctuations drastically increase with decreasing $g^*$ in the range of the single bluff-body pattern. Note that the lift fluctuations for the outer cylinders are larger than that for the central one in the range of $0.3<g^*\lesssim 2$ and *vice versa* in the range of $g^*<0.3$.

## 4. Conclusions

In the present study, we have numerically investigated flow over three circular cylinders in a side-by-side arrangement, which is more representative of flows over multiple bluff bodies of engineering interests than flow over two side-by-side cylinders. For the study, numerical simulations were performed, using the immersed boundary method (Kim et al. 2001), on the flow in the range of $g^*<5$ at $Re=100$. All the flows considered in the present study were assumed to be two-dimensional and laminar even if they were not physically so for small gap spacings (about $g^*<0.4$) at this Reynolds number. From numerical simulations, the following conclusions can be derived :

(1) Flow over three side-by-side cylinders significantly depended on the gap spacing, and a total of five kinds of wake patterns were identified over the flow range: modulation-synchronized ($g^*\gtrsim 2$), inphase-synchronized ($g^*\approx 1.5$), flip-flopping ($0.3<g^*\lesssim 1.2$), deflected ($g^*\approx 0.3$), and single bluff-body wake patterns ($g^*<0.3$).

(2) Modulation occurred because the vortex-shedding frequency for the central cylinder was slightly larger than those for the two outer ones in the range of the modulation-synchronized pattern. On the other hand, the parallel and symmetric modes might occur and their occurrence probability strongly depended on the gap spacing in the range of the flip-flopping pattern.

(3) The bifurcation phenomenon found in the two-cylinder configuration (Kang, 2003) was not observed in the flow over three side-by-side cylinders.

(4) The flow statistics, such as the shedding frequency and drag and lift coefficients, strongly depended on the gap spacing and there was no

biased direction between the two outer cylinders (except for the deflection pattern) in the sense of time averaging.

(i) There existed mean repulsive lift forces exerted in the two gaps between two adjacent cylinders all over the gap-spacing range.

(ii) The central cylinder experienced much higher mean drag than the two outer ones at $g^* < 1$, but smaller rms value of the lift-coefficient fluctuations at $0.3 < g^* \lesssim 2$.

(iii) Overall, the variation of the flow statistics with the gap spacing was similar to the two-cylinder case (Kang, 2003).

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